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ELASTIC WAVE PROPAGATION IN TWO-COMPONENT MEDIA*

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The problem of pressure and stress perturbation propagation after the application of a sudden load to the boundary of an elastic porous half-space saturated by a viscous fluid or gas is investigated. The solution is represented in the form of integrals over segments connecting singularities of the Fourier transform in time for the solution of the problem.

Solutions of non-stationary problems within the framework of the model of a two-component porous medium saturated by a viscous fluid /1—4/ are ordinarily constructed with certain constraints on the values of the parameters of the multicomponent medium. For instance, these are the softness of the porous medium (weakly-cemented mountain rock) /3/, the smallness of the coefficient characterizing the dissipative properties of the medium (slightly viscous saturating fluid) /5/, etc.

The problem of longitudinal wave propagation in a porous medium saturated by a viscous fluid or gas is investigated below for arbitrary values of the parameters of this system and a complete analytic solution is constructed that is suitable for any value of the time the process takes.

1. We consider the problem of pressure and stress perturbation propagation after a sudden application of a load to the boundary of an elastic half-space saturated with a viscous fluid, or (as is equivalent in a mathematical sense), to the bottom hole of a drainage gallery stripping strata with absolutely rigid roofs and basements /3/. The system of equations describing such motion has the form /1—4/

$$P\frac{\partial^2 u}{\partial z^2}+Q\frac{\partial^2 v}{\partial z^2}=\rho_1\frac{\partial^2 u}{\partial t^2}+l(u-v) \tag{1.1}$$
$$R\frac{\partial^2 v}{\partial z^2}+Q\frac{\partial^2 u}{\partial z^2}=\rho_2\frac{\partial^2 v}{\partial t^2}-l(u-v)$$
$$(l=\beta_1\partial/\partial t+\beta_2\partial^2/\partial t^2,\ \rho_1=(1-f)\rho_{10},\ \rho_2=f\rho_{20})\,.$$

Here f is the porosity, $\partial u/\partial t$, $\partial v/\partial t$ are the rates of displacement of the skeleton and the fluid, ρ_{10}, ρ_{20} are the solid phase and fluid densities, β_1 is a coefficient characterizing the dissipative properties of the medium ($\beta_1=\eta f^2/\delta$, η is the viscosity, and δ is the permeability), β_2 is the coefficient of dynamic coupling between the skeleton and the fluid (it was assumed that $\beta_2=0$ in /1, 3/), to be specific we consider $\beta_2=(\rho_1-\rho_2)10^{-3}$; P, Q, R are the moduli of the porous medium /4/ for whose calculation the porosity f, the solid phase C_{10}, fluid C_{20}, and skeleton C_1 compressibilities, and the skeleton shear modulus μ_1 must be given.

The normal stress σ in the solid phase and the pressure p within the fluid have the form /4/

$$\sigma=P\frac{\partial u}{\partial z}+Q\frac{\partial v}{\partial z},\quad -p=Q\frac{\partial u}{\partial z}+R\frac{\partial v}{\partial z}\,. \tag{1.2}$$

The solution of system (1.1) is constructed in the domain $z>0$, $-\infty<t<\infty$ for the following two kinds of boundary conditions:

$$\sigma(z,t)|_{z=0}=0,\ p(z,t)|_{z=0}=p_*H(t),\ -\infty<t<\infty \tag{1.3}$$

or

$$\sigma(z,t)|_{z=0}=\sigma_*H(t),\ p(z,t)|_{z=0}=0,\ -\infty<t<\infty \tag{1.4}$$

that describe the pressure rise in the gallery (liquid piston) or the load application from a highly-permeable piston, respectively, and under the casuality conditions /6, 7/

$$u(z,t)\equiv v(z,t)\equiv 0,\ t<0,\ z>0 \tag{1.5}$$
$$H(t)=0 \text{ when } t<0,\ H(t)=1 \text{ when } t>0\,.$$

2. We use the complex Fourier integral transform in time with the transformation parameter ω to solve the problems (1.1), (1.3), (1.5) and (1.1), (1.4), (1.5). Then the stress field and pressure can be represented by three functions $G_1(\xi,\tau,0)$, $G_1(\xi,\tau,1)$, $G_2(\xi,\tau,0)$ in the dimensionless coordinate ξ and the dimensionless time τ

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$$\frac{p(\xi,\tau)}{p_*} = \sum_{n=0}^{1} \varphi_{1n} G_1(\xi,\tau,n) + \varphi_{20} G_2(\xi,\tau,0) \qquad (2.1)$$

$$\frac{s(\xi,\tau)}{p_*} = \sum_{n=0}^{1} \psi_{1n} G_1(\xi,\tau,n)$$

for the first kind of boundary conditions (1.3), and

$$\frac{p(\xi,\tau)}{s_*} = \sum_{n=0}^{1} f_{1n} G_1(\xi,\tau,n) \qquad (2.2)$$

$$\frac{s(\xi,\tau)}{s_*} = \sum_{n=0}^{1} q_{1n} G_1(\xi,\tau,n) + q_{20} G_2(\xi,\tau,0)$$

for the second kind of boundary conditions (1.4). Here

$$G_j(\xi,\tau,n) = \frac{1}{2\pi} \int_{-\infty+i\sigma^\circ}^{+\infty+i\sigma^\circ} G_j^F(\xi,\chi,n) e^{-i\chi\tau} d\chi, \qquad \sigma^\circ > 0 \qquad (2.3)$$

$$G_j^F(\xi,\chi,n) = G_{j1}^F(\xi,\chi,n) + (-1)^j G_{j2}^F(\xi,\chi,n)$$

$$G_{jm}^F(\xi,\chi,n) = \frac{\exp(i\gamma_m(\chi)\xi)}{(-i\chi)^{n+1}} \left[\delta_{j2} + \delta_{j1} \frac{k_6\chi}{\Omega(\chi)}\right]$$

$$\delta_{jm} = 1,\ j = m;\ \delta_{jm} = 0,\ j \neq m;\ j = 1, 2;\ m = 1, 2;\ n = 0, 1$$

$$\gamma_m(\chi) = (\chi/2)^{1/2} [\chi a - ib - (-1)^m \Omega(\chi)]^{1/2},\ m = 1, 2$$

$$\Omega(\chi) = [(\chi a + ib)^2 - 4\chi(\chi c + id)]^{1/2} = k_6 (\chi - \chi_+)^{1/2} (\chi - \chi_-)^{1/2}$$

$$\chi_\pm = -i\chi_2^\circ \pm \chi_1^\circ,\ \chi_1^\circ = \varepsilon_1 \chi_{10}^\circ,\ \chi_2^\circ = \varepsilon_1 \chi_{20}^\circ$$

$$\chi_{10}^\circ = 2k_5 k_4^{-1},\ \chi_{20}^\circ = k_3 k_4^{-1},\ a = (1 + M^2 + \varepsilon_2 k_2) k^{-1}$$

$$b = \varepsilon_1 b_0,\ c = [\varepsilon_2 (N - N^{-1}) + 1] k^{-1},\ d = \varepsilon_1 d_0$$

$$b_0 = k_2 k^{-1},\ d_0 = (N + N^{-1}) k^{-1},\ k = M^2 - \varepsilon_3^2$$

$$k_1 = (M^2 - 1)(M^2 N - N^{-1}) + 2\varepsilon_3 (M^2 + 1) + 2\varepsilon_3^2 (N + N^{-1})$$

$$k_2 = k_7 - k_8,\ k_3 = k_1 - \varepsilon_2 k_2^2,\ k_4 = (M^2 - 1)^2 + 2\varepsilon_2 k_1 + \varepsilon_2^2 k_2^2 + 4\varepsilon_3^2$$

$$k_5 = |M^2 - 1 - \varepsilon_3 (N - N^{-1})| k^{1/2},\ k_6 = k_4^{1/2} k^{-1},$$

$$k_7 = M^2 N + \varepsilon_3$$

$$k_8 = \varepsilon_3 + N^{-1},\ \varphi_{10} = -q_{10} = [\varepsilon_2 (M^2 N - N^{-1}) + M^2 - 1] k_4^{-1/2}/2,$$

$$\varphi_{11} = -q_{11} = \varepsilon_1 (M^2 N - N^{-1}) k_4^{-1/2}/2,\ \varphi_{20} = q_{20} = 1/2$$

$$f_{10} = [\varepsilon_2 k_8 + \varepsilon_3 N^{-1}] k_4^{-1/2},\ f_{11} = \varepsilon_1 k_8 k_4^{-1/2}$$

$$\psi_{10} = \varepsilon_2 k_7 + \varepsilon_3 N,\ \psi_{11} = \varepsilon_1 k_7 k_4^{-1/2}$$

$$\varepsilon_1 = \beta_1 h c_2^{-1} (\rho_1\rho_2)^{-1/2},\ \varepsilon_2 = \beta_2 (\rho_1\rho_2)^{-1/2},\ \varepsilon_3 = Q (RN)^{-1}$$

$$c_1 = \left(\frac{P}{\rho_1}\right)^{1/2}, \quad c_2 = \left(\frac{R}{\rho_2}\right)^{1/2}, \quad M = \frac{c_1}{c_2}, \quad N = \left(\frac{\rho_1}{\rho_2}\right)^{1/2}$$

$$\tau = \frac{tc_2}{h}, \quad \xi = \frac{z}{h}, \quad \chi = \frac{\omega h}{c_2}$$

h is a characteristic linear dimension which can, for instance, be selected in the form $h = c_2\rho_2\beta_1^{-1}$ for $\beta_1 \neq 0$.

The functions $G_j(\xi,\tau,n)$ $(j = 1, 2;\ n = 0, 1)$ can be represented as follows in the form of integrals over segments connecting the branch points $\chi = \chi_\pm$, $\chi = 0$, $\chi = -idc^{-1}$ of the functions $\gamma_1(\chi)$, $\gamma_2(\chi)$ whose analytic properties are studied in /8, 9/:

$$G_j(\xi,\tau,n) = [H(\tau_-) - H(\tau_+)] I_j(\xi,\tau,n) + H(\tau_+) I_j^1(\xi,\tau,n) + (-1)^j H(\tau_-) I_j^2(\xi,\tau,n) \qquad (2.4)$$

$$I_j(\xi,\tau,n) = \frac{1}{\pi} \exp(-\chi_2^\circ\tau) \int_{\delta_j}^{\chi_1^\circ} [\exp(-\xi\alpha_+)\Psi_{jn}^{+} + \exp(-\xi\alpha_-)\Psi_{jn}^{-}] dx$$

$$\Psi_{10}^{\pm} = -l_0^{-1} \cos\varphi_\pm, \qquad \Psi_{11}^{\pm} = (\chi_2^0 \cos\varphi_\pm + x \sin\varphi_\pm) l_0^{-1} k_0^{-2}$$

$$\Psi_{20}^{\pm} = \pm l_0 \Psi_{11}^{\pm}, \qquad l_0 = [(\chi_1^\circ)^2 - x^2]^{1/2}, \qquad k_0 = [(\chi_2^\circ)^2 - x^2]^{1/2}$$

$$\varphi_\pm = \xi\gamma_\pm - \tau x, \quad \gamma_\pm = 2^{-1/2}[(Z^\circ + X_\pm^\circ)^{1/2}K_+ + (Z^\circ - X_\pm^\circ)^{1/2}K_-]$$
$$K_\pm = (k_0 \pm \chi_2^\circ)^{1/2}, \quad \alpha_\pm = 2^{-1/2}[(Z^\circ + X_\pm^\circ)^{1/2}K_- - (Z^\circ - X_\pm^\circ)^{1/2}K_+]$$
$$Z^\circ = [(X_\pm^\circ)^2 + (Y^\circ)^2]^{1/2}, \; X_\pm^\circ = -\chi_2^\circ a + b \pm k_6 l_0, \; Y_0 = xa$$
$$I_j^m(\xi,\tau,n) = \frac{1}{\pi}\Big[(-1)^j \delta_{m2} \int\limits_{\delta_1}^{dc^{-1}} \frac{e^{-\tau x}}{(-x)^{n+1}}\theta_{jn}\sin(\xi\sqrt{Ax})\,dx +$$
$$\delta_{m1}\int\limits_{\delta^\circ}^{\delta_2} \frac{e^{-\tau x}}{(-x)^{n+1}}\theta_{jn}\sin(\xi\sqrt{Ax})\,dx\Big] - (-1)^j d_{jn}, \quad m = 1, 2$$
$$k^\circ = [(\chi_2^\circ - x)^2 + (\chi_1^\circ)^2]^{1/2}, \; \theta_{11} = \theta_{10} = -x/k^\circ, \; \theta_{20} = 1$$
$$A = (k_6 k^\circ + b - xa)/2, \; c_\pm = 2^{1/2}[a \pm (a^2 - 4c)^{1/2}]^{-1/2}$$
$$\tau_\pm = \tau - \xi/c_\pm$$
$$d_{10} = 0, \quad d_{20} = -1, \quad d_{11} = \begin{cases} \operatorname{sign}(\chi_2^\circ)\,k_6 b^{-1}, & |\chi_2^\circ| \geqslant \delta^\circ \\ 2\pi^{-1}k_6 b^{-1}\arcsin(\chi_2^\circ/\delta^\circ), & |\chi_2^\circ| \leqslant \delta^\circ \end{cases}$$
$$\delta_1 = \begin{cases} \delta^\circ, & \chi_2^\circ \leqslant \delta^\circ \\ \chi_2^\circ, & \delta^\circ \leqslant \chi_2^\circ \leqslant dc^{-1} \\ dc^{-1}, & dc^{-1} \leqslant \chi_2^\circ \end{cases}$$
$$\delta_2 = \begin{cases} 0, & |\chi_2^\circ| \geqslant \delta^\circ \\ [(\delta^\circ)^2 - (\chi_2^\circ)^2]^{1/2}, & |\chi_2^\circ| \leqslant \delta^\circ \end{cases}.$$

As is seen, δ_1, δ_2 are variable limits of integration that depend on the parameters of the porous medium, where δ° is an arbitrarily small quantity.

The near-front asymptotic forms for $\tau = \xi/c_-$ and the jumps for $\tau = \xi/c_+$ have the following form for the functions $G_j(\xi, \tau, n)$

$$G_j(\xi,\tau,n) = (-1)^j(\tau_-)^n H(\tau_-)\exp(-\xi\eta_-), \quad \tau_- \to 0 \tag{2.5}$$
$$[G_j(\xi,\tau,n)]_+ = (\tau_+)^n \exp(-\xi\eta_+), \; \tau_+ \to 0; \; j = 1,2$$
$$(\eta_\pm = c_\pm \varepsilon_1 (b_0 \pm \chi_{20}^\circ k_6)/4).$$

Their behaviour as $\tau \to \infty$ can be obtained from the representations of the functions $G_j(\xi, \tau, n)$ in the form (2.3), (2.4):

$$G_1(\xi,\tau,0) = \xi k_6 \frac{\tau^{-3/2}}{2\sqrt{\pi b}}, \quad G_1(\xi,\tau,1) = -\xi k_6 \frac{\tau^{-1/2}}{\sqrt{\pi b}} \tag{2.6}$$
$$G_2(\xi,\tau,0) = 2 - \xi\sqrt{\frac{b}{\pi}}\,\tau^{-1/2}.$$

Note that as the coefficient ε_1, characterizing the dissipative properties of the medium, tends to zero, the relationships (2.1), (2.2) can be reduced to the form

$$p/p_* = (\varphi_{20} - \varphi_{10})H_1(\tau) + H(\tau_+), \; \sigma/p_* = -\psi_{10}H_1(\tau) \tag{2.7}$$
$$\sigma/\sigma_* = (g_{20} - g_{10})H_1(\tau) + H(\tau_+), \; p/\sigma_* = -f_{10}H_1(\tau)$$
$$(H_1(\tau) = H(\tau_-) - H(\tau_+))$$

3. The solutions (2.1) and (2.2) yield the following pattern of the process from the representation of the functions $G_j(\xi, \tau, n)$ in the form (2.4) and their behaviour in the neighbourhoods of the characteristic points $\tau = \xi/c_-$, $\tau = \xi/c_+$ and $\tau \to \infty$.

After a sudden application of the pressure on the boundary of the half-space $(\xi = 0)$ at a point at a distance ξ, a pressure wave p/p_* appears in the fluid and a stress wave σ/p_* in the skeleton after a time interval $\tau = \xi/c_-$ with jumps, respectively, of magnitudes

$$[p/p_*]_- = (\varphi_{20} - \varphi_{10})\exp(-\xi\eta_-), \quad [\sigma/p_*]_- = -\psi_{10}\exp(-\xi\eta_-). \tag{3.1}$$

At the time $\tau = \xi/c_+$ the pressure and stress also experience jumps, respectively, by amounts

$$[p/p_*]_+ = (\varphi_{20} + \varphi_{10})\exp(-\xi\eta_+), \quad [\sigma/p_*]_+ = \psi_{10}\exp(-\xi\eta_+). \tag{3.2}$$

With time the pressure in the fluid rises to the value of the applied pressure on the half-space boundary while the stress tends to zero as $\tau \to \infty$:

$$p/p_* = 1 - \varepsilon_1^{1/2}k_7(kk_2\pi)^{-1/2}\xi\tau^{-1/2} \tag{3.3}$$
$$\sigma/p_* = -\varepsilon_1^{1/2}k_7(kk_2\pi)^{-1/2}\xi\tau^{-1/2}.$$

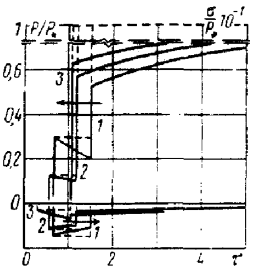

Fig.1

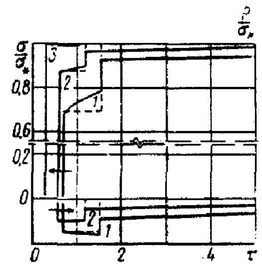

Fig.2

For the second method of applying the load (1.4), the qualitative pattern of stress and pressure perturbation through the point ξ is conserved. The stress and pressure jumps for $\tau = \xi/c_-$ and $\tau = \xi/c_+$ are determined by formulas analogous to (3.1) and (3.2) (with the substitutions $p/p_* \to \sigma/\sigma_*$, $\sigma/p_* \to p/\sigma_*$, $\varphi_{j0} \to g_{j0}$, $\psi_{10} \to f_{10}$). As $\tau \to \infty$ we have formulas analogous to (3.3) (with the substitution $p/p_* \to \sigma/\sigma_*$, $\sigma/p_* \to p/\sigma_*$, $k_7 \to k_8$).

The magnitudes of the jumps $\tau = \xi/c_-$ and $\tau = \xi/c_+$ diminish by a factor of e, respectively, at the distances $l_- = 1/\eta_-$, $l_+ = 1/\eta_+$, which are inversely proportional to ε_1 (i.e., the viscosity), and they are not substantial at large distances.

4. Computations were performed for certain models. The physical constants of quartz sandstone, oil, gas, and water were taken from /10/.

Changes in the stresses σ/p_*, σ/σ_* and the pressures p/p_*, p/σ_* in a water-saturated (curve *1*), oil-saturated (curve *2*), and gas-saturated (curve *3*) half-space are shown in Figs.1 and 2 as a function of the time τ in a section at a distance $\xi = 1$ from the surface of application of the pressure p_* and the stress σ_*. The quantities calculated by means of (2.1), (2.2), (2.4) correspond to the solid lines, and quantities calculated by means of (2.7) for the fluid viscosity $\varepsilon_1 = 0$, by dashes. The dimensionless distances of the jump penetration are for the gas-saturated $l_- = 2.3\cdot10^3$, $l_+ = 2.1$, oil-saturated $l_- = 4.2\cdot10^2$, $l_+ = 1.6$, and water saturated $l_- = 3.8\cdot10^1$, $l_+ = 1.3$ half-space.

The significant influence of the parameter ε_1 on the stress and pressure distribution in a porous medium follows from the results shown in Figs.1 and 2.

REFERENCES

1. FRENKEL' YA.I., On the theory of seismic and seismoelectric phenomena in moist soil, Izv. Akad. Nauk SSSR, Ser. Geogr. Geofiz., Vol.8, No.4, 1944.
2. BIOT M., Theory of the propagation of elastic waves in fluid-saturated porous solids. 1. Low-frequency range, J. Acoust Soc. Amer., Vol.28, No.2, 1956.
3. NIKOLAEVSKII V.N., BASNIEV K.S., GORBUNOV A.T. and ZOTOV G.A., Mechanics of Saturated Porous Media. Nedra, Moscow, 1970.
4. KOSACHEVSKII L. YA., On elastic wave propagation in two-component media, PMM Vol.23, No.6, 1959.
5. RAKHMATULIN KH.A., SAATOV YA., FILIPPOV N.G. and ARTYKOV T.U., Waves in Two-Component Media. Fan, Tashnkent, 1974.
6. FELSEN L. and MARCUVITZ N., Wave Radiation and Scattering, Vol.1, Mir, Moscow, 1978.
7. MOKRIK R.I. and BARAN V.P., On radiation conditions in dynamic problems of elasticity theory. Dokl. Akad. Nauk UkrSSR, Ser. A., No.8, 1977.
8. MOKRIK R.I. and PYR'EV YU.A., Certain questions of the theory of conjugate fields in the mechanics of continuous media. Dokl. Akad. Nauk Ukr.SSR, Ser. A, No.3, 1981.
9. MOKRIK R.I. and PYR'EV YU.A., Analytic properties of characteristic parameters of generalized thermoelasticity, Matematich. Metody i Fiziko-mekhanich. Polia, vyp,. 15, Naukova Dumka, Kiev, 1982.
10. LYAKOVITSKII F.M. and RAPOPORT L.I., Application of the Frenkel-Biot theorem to the calculation of the elastic wave velocity and absorption in saturated porous media. Priklad. Geofizika, vyp. 66, Nedra, Moscow, 1972.

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